Traffic characteristics of a branching Poisson input finite-capacity Queue

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Abstract

In a telecommunication network system, a branching Poisson input finite-capacity BPP/GI/1/m queue is often appeared. We have evaluated the queue overflow rate and the waiting overtime rate of these schemes. We started with　BPP/M/1/m queue with the distribution of number of children following their parents. We have also evaluated the queue overflow rate and the waiting overtime rate of priority queuing system which consists of BPP/M/1/m queue with the distribution of number of children and Poisson input queue.

Key Word: communication network, queue overflow rate, waiting overtime rate, BPP/M/1/m model, distribution of number of children, priority queuing system

1. Introduction

A branching Poisson input finite-capacity BPP/GI/1/m queue is often appeared in telecommunication network systems. For example, in VoIP (Voice over Internet Protocol) network voice calls are assumed to be generated according to the Poisson process and RTP (Real Time Packet Protocol) voice packets in each call are generated cyclically with an almost constant time span. So this process is regarded as the BPP, in which the first arriving RTP packet in individual call is referred to as ‘parent’ and other packets in each call are referred to as ‘children’. Besides, the number of children is not constant for each call. Namely this process is regarded as BPP with different number of children. In the IP (Internet Protocol) network where both data packets which form a Poisson process and voice packets which form a Branching Poisson Process are added, the voice packets are required to have a higher priority to reduce the transmission delay time and guarantee the quality of voice communication service.

In these telecommunication network systems, the overflow rate and the waiting overtime rate in the queue which is prepared in each network should be evaluated to clarify the communication quality [1]. In order to evaluate the queue overflow rate and the waiting overtime rate applicable to these schemes mentioned above, BPP model with finite capacity queue is to be analyzed. But unfortunately BPP is not renewal, we cannot use the standard theory of the GI/GI/1 queue where GI stands for a renewal process [2]-[6].

Recently, we have shown that a diffusion approximation technique is applied to obtain the mean performance measures (mean number of customers in the system, and the mean waiting time) in a branching Poisson input finite-capacity BPP/GI/1/m queue; see [7]. This technique handles only the process where the number of children is constant for all parents. BPP/M/1/m queue with the distribution of number of children or BPP/M/1/m queue with higher priority than the Poison input queue has not been handled in this technique. Besides, traffic characteristic of these branching Poisson input finite-capacity queue has not yet been clarified.

The main purpose of this paper is to obtain the queue overflow rate or the waiting overtime rate for the BPP/M/1/m queue with the distribution of number of children or BPP/M/1/m queue with higher priority than the Poison input queue in order to clarify the fundamental traffic characteristics for these queuing systems.

2. BPP/M/1/m Queue with the Distribution of Number of Children

2.1 Evaluation Model

In VoIP (Voice over Internet Protocol) network, voice calls are assumed to be generated according to the Poisson process. And RTP (Real Time Packet Protocol) voice packets, which include digitalized voice information, are forwarded cyclically after a constant time span from the time that the previous RTP packet has been forwarded. In these forwarding schemes a RTP packet arrives at the queue after a constant time span from the time that the previous RTP packet has been arrived. Besides, the number of RTP packets is not constant for each call. Namely, these processes of RTP packets forwarding are regarded to be a Branching Poisson Process (BPP) with the distribution of number of children. The first arriving RTP packet in individual call is referred to as ‘parent’ and other RTP packets are referred to as ‘children’. The parents are assumed to form a Poisson Process. So in order to evaluate the queue overflow probability and the waiting overtime rate applicable to these schemes, BPP/GI/1/m queue with the distribution of number of children is to be analyzed. But we cannot obtain the standard theory or approximate method for the performance measures of this BPP/GI/1/m queue. So, we have also evaluated the performance measures of this BPP/GI/1/m queue via simulation.

2.2 Simulation Program

We developed a simulation program [8] that simulates the following procedures: (1) Customers arrive at the queue according to the Poisson process and are registered in the queue if the queue is not full. (2) These customers are also registered in the reforwarding queue and the reforwarding timer starts. Children in the BPP are registered in the sending queue in this process. (3) The processing server serves the customer registered in the queue one by one. (4) Customers are reforwarded from the reforwarding queue and registered in the queue if the queue is not full when the reforwarding timer is expired. Children in the BPP are generated in this process.

In this simulation program control functions such as customer arrival or service execution is generated only when the state transition, such as the customer is generated or the service starts, occurs. This

本論文冒頭

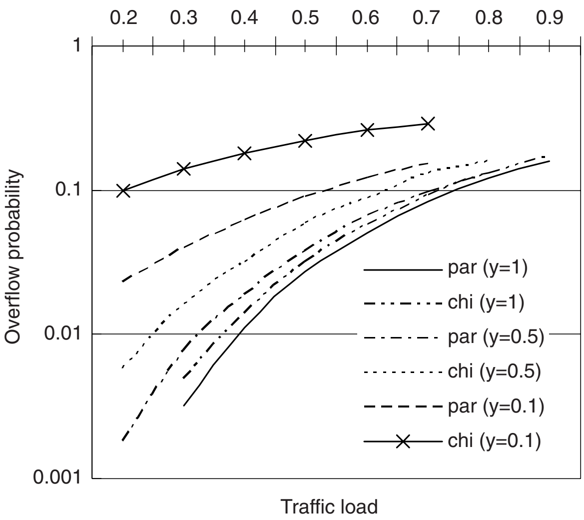
enables the program to achieve high speed simulation. This simulation program has been written by programming language C

2.3 Evaluation Results

Examples of the evaluated over flow probability of parents or children versus the traffic load (λ) for the intervals between two adjacent points of 0.1,0.5 or 1 (y=0.1,0.5,1), assuming the waiting rooms of 4 (m=4), a 0.1 mean service time (s=0.1), a 10 number of children (k=10) are shown in Fig. 1. Here, the intervals between two adjacent points means that the duration from the time that a parent or child arrives at the queue to the time that next child arrives at the queue, and is denoted by y,

In this evaluation the over flow probability for the children is obtained from the average of the over flow probability of 10 children. The Poisson distribution is adopted for the distribution of the number of children in these evaluations. The evaluation results show that the overflow probability of children is larger than the overflow probability of parents for all case of y. The evaluation result also shows that y becomes larger the difference of the over flow probability of parents and the over flow probability of children becomes smaller. This is because when y becomes large enough compared with the mean service time total packets arrival of parents and children almost forms Poisson process.

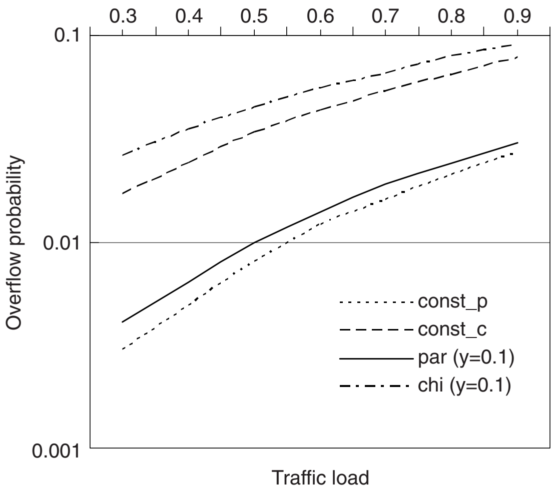
Fig.1 Overflow probability vs traffic load



Examples of the evaluated over flow probability of parents or children versus the traffic load (λ) for the distribution of the number of children or the constant number of children, assuming the intervals between two adjacent points of 0.1 (y=0.1), the waiting rooms of 4 (m=4), a 0.1 mean service time (s=0.1), a number of children of 3 (k=3) are shown in Fig. 2. Numerical results of this evaluation show that the overflow probability of parents or children in the case of the distribution of number of children is larger than the overflow probability in the case of constant number of children.

Examples of the evaluated over flow probability of parents or children versus the traffic load (λ) for the distribution of number of children or the constant number of children, assuming the intervals between two adjacent points of 0.5 (y=0.5), the waiting rooms of 4 (m=4), a 0.2 mean service time (s=0.2), a number of children of 3 (k=3) are shown in Fig. 3. When the intervals between two adjacent points becomes large enough compared with the mean service time, the overflow probability of parents or children in the case of the distribution of number of children becomes almost the same as the overflow probability in the case of constant number of children. This is because the intervals between two adjacent points become large enough compared with the mean service time, the Poissonian arrivals is expected to be assumed both in the case of the distribution of number of children and in the case of constant number of children.

Fig.2 Overflow probability versus Traffic load



Examples of the evaluated over flow probability of parents or children versus the traffic load (λ), assuming the intervals between two adjacent points of 0.1 or 0.5 (y=0.1,0.5), the waiting rooms of 4 (m=4), a 0.1 mean service time (s=0.1), a number of children of 10 (k=10) are shown in Fig. 4.

Numerical results of these evaluation show that when the number of children becomes larger, the overflow probability in the case of distribution of the number of children is almost the same as the overflow probability in the case of constant number of children.

This is because when the number of children becomes large enough, traffic characteristics is determined only by the number of children, so influence of the distribution of the number of children to the overflow probability can be ignored.

Fig. 5, 6 show examples of the waiting time distribution. These curves show the cumulative distribution obtained by 1-(waiting overtime probability) for the interval between two adjacent points of 0.5

Fig.3 Overflow probability versus traffic load

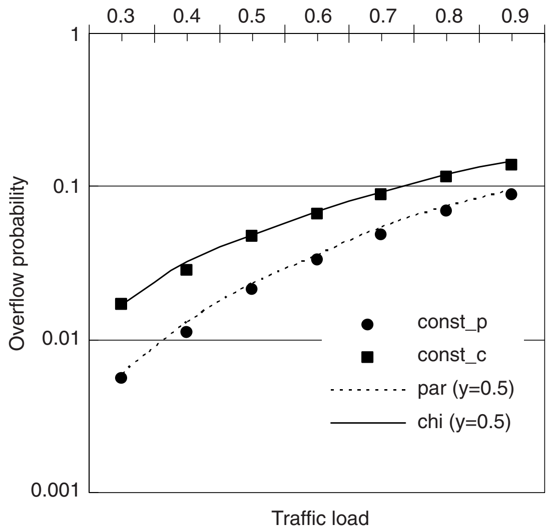
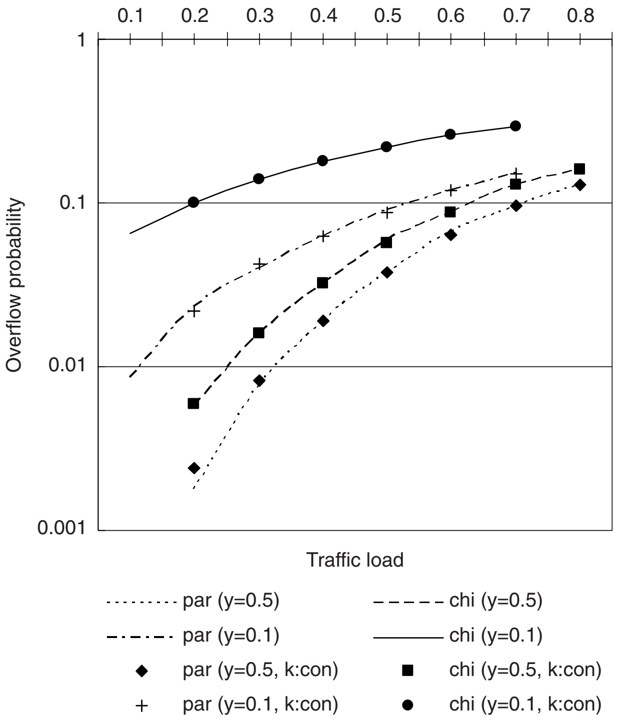


Fig.4 Overflow probability vs traffic load (k=10)



本論文冒頭

Fig.5 Cumulative probability (k=3)

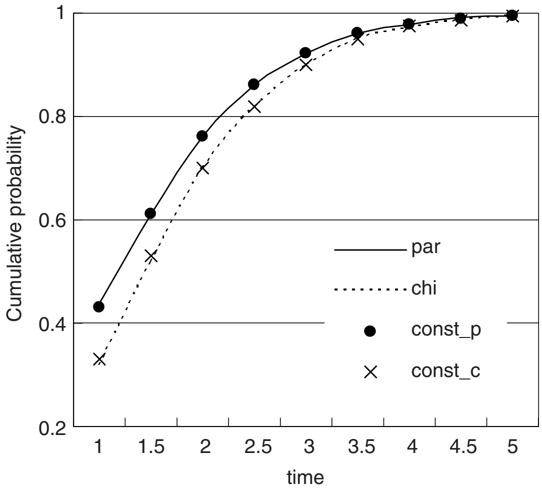
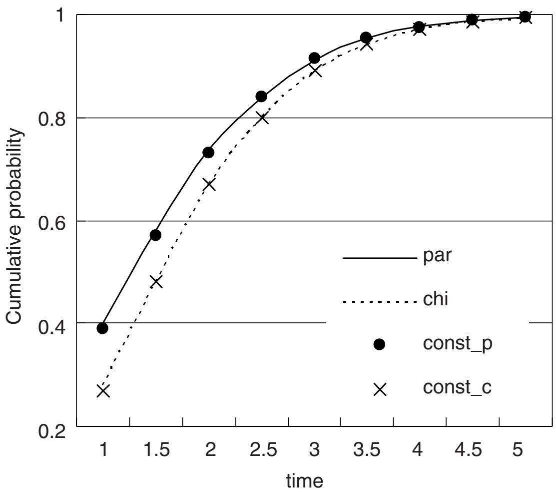


Fig.6 Cumulative probability (k=10)



(y=0.5), the waiting rooms of 4 (m=4), a 0.5 mean service time (s=0.5), a number of children of 3 or 10 (k=3,10), a 0.5 traffic load (λ =0.5). Numerical results of these evaluation show that the cumulative distribution in the case of distributed number of children is almost same as the cumulative distribution in the case of constant number of children both in the case of k=3 and k=10.

From these evaluation results we can say that when the intervals between two adjacent points becomes large enough compared with the mean service time or the number of children is large enough, the overflow probability or the waiting time distribution of parents or children in the case of the distribution of number of children can be approximated by the evaluation techniques for the case of constant number of children.

3. Priority Models

3.1 Evaluation Model

Priority is given to a certain class of traffic to improve the grade of service over other classes. In the IP network where both data packets which form a Poisson process and voice packets which form a Branching Poisson Process are added and waiting queues are prepared for both classes, the voice packets are required to have a higher priority to reduce the transmission delay time and guarantee the quality of voice communication services. In this model, when waiting packets exist in both queue for the voice packets and queue for the data packets, every time a packet in service terminates the higher priority packet waiting for service enters service. A higher priority packet does not interrupt a lower priority packet in service. (non-preemptive priority model).

To form the priority in this queuing system following two control methods are considered, (1) lower priority packets are served after all higher priority packets waiting for services are served, (2) lower priority packets are served after a fixed number of higher priority packets waiting for services are served.

3.2 Evaluation Results

Examples of the evaluated over flow probability of higher priority packets (parents or children) and lower priority packets versus n, which is the number of higher priority packets served before lower priority packets are served in the control method (2), assuming a 0.2 traffic load of higher priority parents, a 0.6 traffic load of lower priority packets, a 2 interval between two adjacent point of higher priority packets (y=2), the waiting rooms of 4 (m=4), a 0.5 mean service time (s=0.5), a number of children of 2 (k=2) are shown in Fig. 7. Total traffic load of BPP (the sum of parents and children) is equal to the traffic load of Poisson traffic. In this figure evaluation results for the control method (1) are also shown as black circles. These evaluation results show that in the case of n<3 overflow probability of higher priority (BPP) packets is larger than that of lower priority (Poisson) packets because of a bursty nature of BPP traffic in spite that the total traffic load of BPP is equal to the traffic load of Poisson traffic. So in order to keep the quality of BPP packets better than that of Poisson packets, n has to be larger than 3 in this evaluation environment.

Examples of the evaluated overflow probability of higher priority packets and lower priority packets, assuming a 0.2 traffic load of higher priority parents, a 0.6 traffic load of lower priority packets, a 2 interval between two adjacent point of higher priority packets (y=2), the waiting rooms of 4 (m=4), a 0.5 mean service time (s=0.5), a number of children of 2 (k=2) are shown in Fig. 8. In these evaluation total traffic load of BPP (the sum of parents and children) is equal to the traffic load of Poisson traffic, and the control method (1) mentioned above is adopted to form the priority in this queuing sys-

Fig.7 Overflow probability vs the number of higher priority packets served before lower priority packets are served (n)

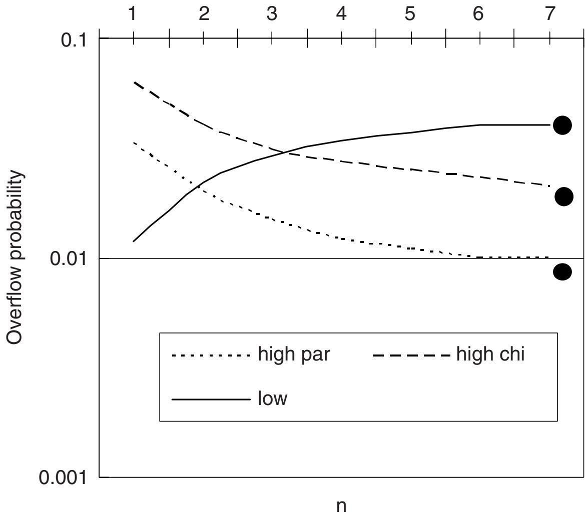
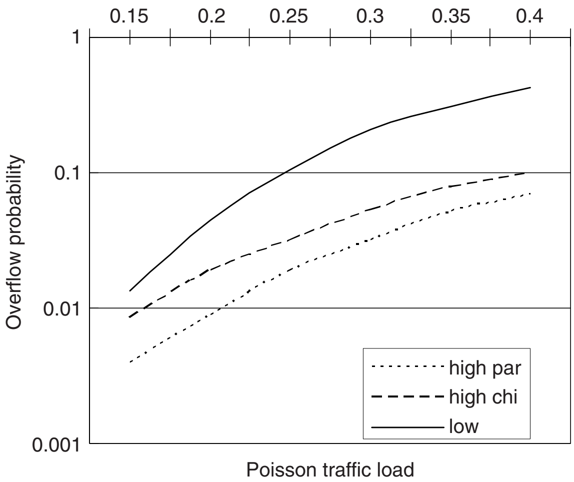


Fig.8 Overflow probability versus Poisson traffic

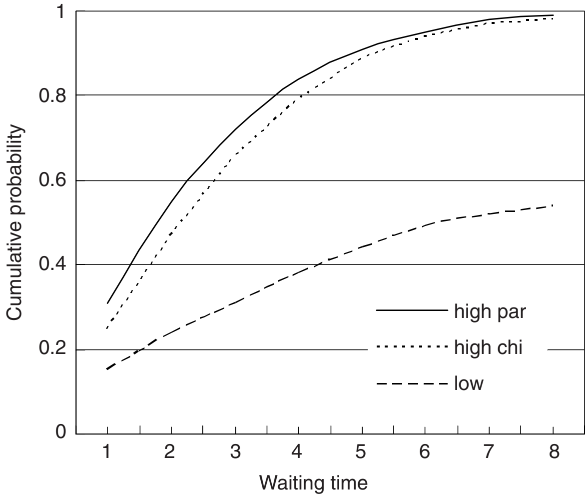


tem. From these evaluation results we can say that the overflow probability of the lower priority packets becomes large more rapidly than that of higher priority packets when the traffic load becomes large.

Fig.9 show the cumulative distribution for the interval between two adjacent points of 2 (y=2), the waiting rooms of 4 (m=4), a 1 mean service time (s=1), a number of children of 2 (k=2), a 0.3 traffic load of higher priority parents and a 0.9 traffic load of lower priority packets. To form the priority in this queuing system the control method (1) mentioned above is also adopted. From these evaluation

本論文冒頭

Fig.9 Cumulative probability (k=2)



results we can say that (1) difference between the cumulative probability of the parents and that of children of higher priority packets is small, (2) difference between the cumulative probability of higher priority packets and that of lower priority packets is very large, (3) the cumulative probability of the lower priority packets distributes over wide range of waiting time.

4. Conclusion

We have evaluated the queue overflow rate and the waiting overtime rate of BPP/M/1/m queue with the distribution of number of children and priority queuing system which consists of BPP/M/1/m queue with the distribution of number of children and Poisson input queue. From these evaluations we have presented that the overflow probability of parents or children in the case of the distribution of number of children is larger than the overflow probability in the case of constant number of children. But when the intervals between two adjacent points becomes large enough compared with the mean service time or the number of children is large enough, the overflow probability of parents or children in the case of the distribution of number of children can be approximated by the evaluation techniques for BPP/M/1/m with constant number of children. We also clarified that we can obtain the number of higher priority packets served before lower priority packets are served in order to guarantee the quality of services for higher priority packets.

From these evaluation results we have clarified the fundamental traffic characteristic for the BPP/M/1/m queue with the distribution of number of children or BPP/M/1/m queue with higher priority than the Poison input queue.

For further study we intend to perform an approximation analysis of queue overflow probability, mean waiting time, waiting time distribution etc. for these BPP queuing models mentioned above, such as BPP/M/1/m queue with the distribution of number of children and non-preemptive priority

本論文冒頭

model.

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